11-2 Arithmetic Series

Series is the sum of a sequence

8, 12, 16, 20, 24, sequence

8 + 12 + 16 + 20 + 24 series

$S_1 =$ 8

$S_2 =$ 8+12 = 20

$S_3 =$ 36

$S_4 =$ 56

$S_5 =$ 80

In general, $a_1, a_2, a_3, a_4, \ldots a_n$

$S_n = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

Σ sigma summation sign

8 + 12 + 16 + 20 + 24

Find a_n. $a_n = a_1 + (n-1)d$

$a_n = 8 + (n-1)4$

$a_n = 4n + 4$

$\sum_{n=1}^{5}(4n + 4)$ ← a_n → simplified

"the sum of 4n + 4 as n goes from 1 to 5"

Expanded form = $8+12+16+20+24$

$\sum_{n=3}^{7}(2n - 3)$

Put in Expanded form.

$= 3 + 5 + 7 + 9 + 11$

Put the following series into sigma notation:

3 + 6+ 9 + ... + 36

$a_n = 3 + (n-1)3$

$3 + 3n - 3$

$36 = 3n$

$12 = n$

$\sum_{n=1}^{12} 3n$

Challenge:

Find the sum of the integers from 1 to 100

$a_1 = 1 \quad a_{100} = 100 \quad 101$

$2 \quad 99 \quad 101$ $\times 50 = 5050$

$3 \quad 98 \quad 101$

$\vdots$

50

$S_n = \frac{n}{2}(a_1 + a_n)$

Friedrich Gauss

a_1 =
a_{100} =

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Ex:
Find the sum of the given sequence:
5, 10, 15, 20

$$S_4 = \frac{4}{2}(5 + 20) \quad (a_1 + a_4)$$
$$= 50$$

Ex:
Find the sum of the given sequence:
3, 6, 9, 12, 15, 18

Ex:
Find the sum of the first 50 terms of the given sequence:
3, 6, 9, 12, 15, 18

$a_{50} = 3 + (49)3$

$S_{50} = \frac{50}{2}(3 + 150)$

$S_{50} = 3825$

When you don't have the last term, either find it or use this formula.

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

Ex:
$a_n = -14$
$n = 9$
$d = -8$
$S_9 =$ 162

$a_n = a_1 + (n-1)d$
$-14 = a_1 + (8)(-8)$
$50 = a_1$

$S_9 = \frac{9}{2}(50 + -14)$

HW p586
15-37odd, 40